Chapter 3
Sections 3.1 & 1.5
Warm-up Problem A. Let $f(x) = x^2 - 100$. What is the y-intercept of the function? What are the x-intercepts?

$(0, -100)$, $(10, 0)$

Problem 1. Find, by factoring, the x-intercepts of the following functions, and write each quadratic function in factored form:

(a) $j(x) = x^2 + 2x + 1$

$(x+1)^2 = 0$

$(-1, 0)$

(b) $f(x) = x^2 - 3x - 10$

$(x-5)(x+3) = 0$

$(5, 0)$ and $(-3, 0)$

(c) $g(x) = 11x - 4 + 3x^2$

$3x^2 + 12x - x - 4$

$3x(x+4) - (x+4) \Rightarrow (3x-1)(x+4) = 0$

$(\frac{1}{3}, 0)$ and $(-4, 0)$

(d) $h(x) = 10x^2 + x - 3$

$10x^2 + 6x - 5x - 3$

$2x(5x+3) - (5x+3) \Rightarrow (2x-1)(5x+3) = 0$

$(\frac{1}{2}, 0)$ or $(-\frac{3}{5}, 0)$

Problem 2. Consider the functions $f(t) = t^2$, $g(t) = t^2 + 2t + 2$, and $h(t) = t^2 + 2t - 2$.

(a) Try finding the t-intercepts of $f(t)$, $g(t)$, and $h(t)$ by factoring. Does anything go wrong?

Notice that

* $g(t)$ and $h(t)$ don't factor

but $f(t)$ does factor

$f(t) = t \cdot t = (t+0)(t+0)$

(b) Find the zeros of $f(t)$, $g(t)$, and $h(t)$ using the quadratic formula, and write $g(t)$ in factored form.

f:

$$\frac{-0 \pm \sqrt{0^2 - 4(1)(0)}}{2(1)} = 0$$

g:

$$\frac{-2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

← negative discriminant

NO zeros

h:

$$\frac{-2 \pm \sqrt{4 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{12}}{2} \Rightarrow x = \frac{-2+\sqrt{12}}{2}, \frac{-2-\sqrt{12}}{2}$$

Problem 3. Find a formula for the quadratic function whose graph has a y-intercept at $y = -8$ and x-intercepts at $x = -4$ and $x = 2$. *Hint: Work backwards in what you have done before.*

$f(x) = (x+4)(x-2) = x^2 + 2x - 8$

Problem 4. For each of the following, find the vertex and the axis of symmetry. Does each parabola open upward or downward?

(a) $f(x) = 0.5(x-3)^2 + 1$

up

V: $(3, 1)$

$x = 3$ axis of sym

(b) $g(x) = -(x+6.1)^2 - 8.5$

down

V: $(-6.1, -8.5)$

$x = -6.1$ a.o.s

Problem 5. Suppose we have the function $f(x) = -x^2 + 4x$. Find the x-intercepts and the y-intercept of this function. What are the domain and range of $f(x)$? Draw a coordinate axis and use this information sketch a graph of $f(x)$.

x-int: $(0,0)$

y-int: $(0,0), (4,0)$

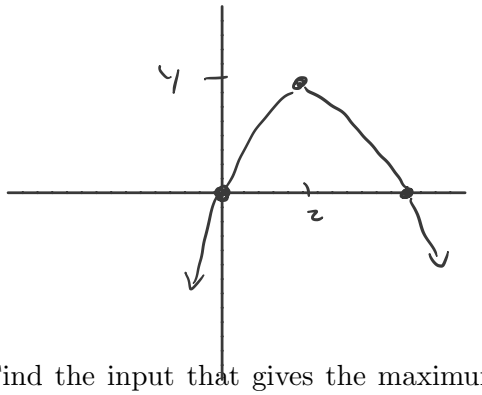
dom: All reals i.e. $(-\infty, \infty)$

rng: $(-\infty, 4]$

$-(x^2 - 4x + 4 - 4)$

$= -(x-2)^2 + 4$

V: $(2, 4)$



Problem 6. For the following functions do each of the following. (i) Find the input that gives the maximum output. (ii) Find the maximum output. (iii) Find the input that gives the minimum output. (iv) Find the minimum output.

(a) $f(x) = (x-1)^2 - 21$

no max

min $\rightarrow x = 1$

$\downarrow$

$y = -21$

(b) $g(x) = 3x - 20 - x^2 \rightarrow -(x - \frac{3}{2})^2 - 20 - (\frac{3}{2})^2$

no min

max $\rightarrow x = \frac{3}{2}$

$\searrow$

$y = -20 - (\frac{3}{2})^2$

Problem 7. Find a formula for a quadratic equation whose graph passes through the point $(11, 11)$ and has its axis of symmetry at $x = 10$. Is there more than one possible formula? What if you know the vertex of the parabola is at the point $(10, 12)$?

$(x-10)^2 + 10$ or $-(x-10)^2 + 12$

$\uparrow$

only one w/ V: $(10, 12)$

Problem 8. Write each of the following quadratic functions in vertex form using any method. Identify the vertex and axis of symmetry in each case.

(a) $m(t) = 5t^2 - 30$

V: $(0, -30)$

axis: $x = 0$

(b) $n(t) = t^2 + 2t + 3$

$(t+1)^2 + 2$

V: $(-1, 2)$

axis $x = -1$

(c) $j(x) = x^2 + 5x + 13$

$\left(x + \frac{5}{2}\right)^2 + 13 - \frac{25}{4}$

V: $\left(-\frac{5}{2}, 13 - \frac{25}{4}\right)$

axis $x = -\frac{5}{2}$

Problem 9. A ball is thrown into the air. Its height, in feet, t seconds later is given by

$$h(t) = 80t - 16t^2.$$

(a) Evaluate and interpret $h(2)$.

$80(2) - 16(4)$

(b) Solve the equation $h(t) = 64$. Interpret your solutions.

2 seconds after the throw

(c) When does the ball hit the ground after it is thrown?

$\frac{80}{16}$ seconds later

(d) Find the vertex of $h(t)$. After how many seconds does the ball attain its maximum height? What is its height at that moment?

$-16\left[\left(t - \frac{80}{2 \cdot 16}\right)^2 - \left(\frac{80}{2(16)}\right)^2\right]$

$t = \frac{80}{2 \cdot 16}$ $h = \left(\frac{80}{2(16)}\right)^2 \cdot 16$

Problem 10. Suppose a farmer has 20 feet of fencing, and he wants to enclose a section of his ground that borders a river. The area enclosed will be a rectangle, but only three sides will be enclosed by the fence as the other side will be bounded by the river. This question will guide you through finding the maximum area the farmer can enclose.

(a) Draw a picture of the situation. Let w and l denote the width and length, respectively, of the enclosed rectangular field.

(b) What is the area of the enclosed region?

$w \cdot l$

(c) Assuming the farmer uses all 20 feet of available fencing, write an equation that gives the perimeter of the enclosed space.

$l + 2w = 300$

(d) In your answer for part (c), one of the variables will be easier to solve for than the other. Solve for this variable.

$l = 300 - 2w$

(e) Plug your result from part (d) into your result in part (b). You should now have a quadratic equation.

$w(300 - 2w)$

(f) Find the vertex of the parabola in part (e). Is this vertex a minimum or a maximum? How do you know?

$-2\left[\left(w - \frac{300}{4}\right)^2 - \left(\frac{300}{4}\right)^2\right]$

(g) What is the maximum area that the farmer can enclose? What values of l and w should he choose to attain this maximum?

$\frac{(300)^2}{2}$